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Tom Leighton

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A $2d - 1$ Lower Bound for 2-Layer Knock-Knee Channel Routing

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Abstract:

In this paper, we describe a 2-point net channel routing problem with density d that requires channel width $2d - 1$ in the 2-layer knock-knee channel routing model. This means that the $(2d - 1)$ -track algorithms of Rivest, Baratz and Miller [9], Bolognesi and Brown [3], Frank [4], Mehlhorn, Preparata and Sarrafzadeh [7] and Berger, Brady, Brown and Leighton [2] are, in some cases, optimal. Thus any improvement of these algorithms must rely on problem features other than density (such as flux [1]), or must make fundamental changes in the wiring model (such as increasing the number of layers [8] or allowing wires to overlap [2, 5]).

This research was supported by DARPA Contract N00014-80-C-0622, Army Contract DAAL-03-86-K-0171, AF Contract OSR-86-0078, the Bantrell Foundation, and an NSF Presidential Young Investigator Award with matching funds from AT&T and IBM.



1. Introduction

Channel routing plays a central role in the development of automated layout systems for integrated circuits. Many layout systems first place modules on a chip or circuit board and then wire together terminals on different modules that should be electrically connected. This wiring problem is often solved by heuristically partitioning the given space into rectangular channels and then assigning to each such channel a set of wires which are to pass through it. This solution reduces a "global" wiring problem to a set of disjoint (and hopefully easier) "local" channel routing problems. For this reason, channel routing problems have been intensively studied for over a decade, and numerous heuristics and approximation algorithms have been proposed.

In most channel routing problems, the *channel* consists of a rectilinear grid of *tracks* (or *rows*) and *columns*. Along the top and bottom tracks are *terminals*, and terminals with the same label form a *net*. A net with r terminals is called an *r -point net*. The smallest net is a *2-point net*. If $r > 2$, we have a *multipoint net*. The channel routing problem is to connect all the terminals in each net using horizontal and vertical wires which are routed along the underlying rectilinear grid. The goal is to complete the wiring using the minimum number of tracks; i.e., to minimize the *width* of the channel. Often, no constraint is placed on the number of columns used at either end of the channel.

Wire segments are physically located on one of the one or more *layers*. Wire segments in different layers can be connected with *contact cuts*, which can be thought of as very short wire segments running through grid points perpendicular to the routing surface. No two wires can change layers in the same space without being connected.

A variety of models have been proposed for channel routing, with differences depending on the number of layers allowed and on the ways in which wires are allowed to interact. One of the most popular models is the knock-knee model proposed by Rivest, Baratz and Miller [9]. In the knock-knee model, wires are allowed to cross or share corners (i.e., knock-knees), but are not allowed to overlap for any distance. In this paper, we will focus on 2-layer knock-knee channel routing problems with only 2-point nets. More formally, we define a *net* $N_i = (p_i, q_i)$ to be an

ordered pair of integers specifying an entry column p_i and an exit column q_i . A net is said to be *rising* if $q_i < p_i$, *falling* if $p_i < q_i$ and *trivial* if $p_i = q_i$. A *channel routing problem* is a collection of n nets such that no two nets have a common entry or exit column. A *solution* to a channel routing problem consists of an integer t and a collection of n wires W_1, \dots, W_n such that W_i enters the grid in the p_i th column of the 0th row and exits the grid in the q_i th column of the $(t + 1)$ st row. Routing is allowed in rows 1 through t of the grid and in any number of columns. Although wires may alternate layers (via contact cuts), they cannot overlap except at crossover or jog points. The *width* of a solution is t , the number of horizontal tracks used to route the wires. The *optimal width* of a channel routing problem is simply the smallest value of t for which there is a solution.

Many algorithms have been discovered for solving 2-layer knock-knee channel routing problems with width $2d - 1$, where d is the density of the channel routing problem. (For example, see [2], [3], [4], [7], and [9].) The *density* of a channel routing problem is the maximum over all $x \in \mathcal{R}$ of the number of nets crossing the vertical cut of the channel at x . A net $N_i = (p_i, q_i)$ is said to *cross* the cut of the channel at x if $p_i < x < q_i$ or $q_i < x < p_i$. For example, the channel routing problem displayed in Figure 1 has density 3.

It is easy to see that the density of a problem is a lower bound on the channel width needed for its solution. Hence, the $(2d - 1)$ -width algorithms of [2, 3, 4, 7, 9] are within a factor of 2 times optimal for any problem. As many practical channel routing problems have solutions with width very close to d , however, it was hoped that the $(2d - 1)$ -width algorithms could be improved (e.g., to produce $\frac{3d}{2}$ -width solutions for problems with density d).

In this paper, we show that such an improvement is not possible. In particular, we construct a 2-point net channel routing problem with density d (for any d) that requires channel width $2d - 1$. Hence the $(2d - 1)$ -width algorithms are optimal in the worst case, and cannot be improved by density considerations alone.

It is worth pointing out that improvements in the performance of the algorithms can be achieved if additional parameters are considered or if different models are used. For example, Baker, Bhatt and Leighton [1] showed that any 2-point net channel routing problem with density d and flux f can be solved using $d + O(f)$ tracks, even if knock-knees are not allowed. (The

definition of f_{uz} is somewhat technical, but roughly corresponds to a horizontal measure of density. In the worst case $f \sim \sqrt{n}$, but f is often a small constant in practical problems.) Alternatively, Preparata and Lipski [8] showed that d tracks are sufficient if 3 layers of wiring are allowed, thus achieving the lower bound for every problem. More recently, Berger, Brady, Brown and Leighton showed that $d + O(\sqrt{d})$ tracks are sufficient using 2 layers if unit-length vertical wire segments are allowed to overlap other vertical wire segments. (For a more complete listing of other bounds and algorithms for channel routing, we refer the reader to [6].)

The paper is divided into 6 sections. In Section 2, we construct the density- d lower bound example, and in Section 3, we prove that it requires $2d - 1$ tracks. We conclude in Sections 4-6 with some remarks, acknowledgements and references.

2. The Lower Bound Construction

Although it may seem somewhat complicated at first, our lower bound construction is quite simple. The problem consists primarily of trivial nets along with a few falling nets arranged as widely-spaced 1-shifts. For example, an informal illustration of the construction for $d = 2$ is shown in Figure 2.

More formally, our hard density- d channel routing problem $R_{d,n}$ is composed of nets N_1, N_2, \dots, N_n where $N_i = (i, q_i)$ and

$$q_i = \begin{cases} i + 4^s s!^2 & \text{if } i \equiv 4^{s-1} \pmod{4^s s!^2} \text{ for some } s \leq d \\ i & \text{otherwise} \end{cases}$$

for $1 \leq i \leq n$ and $n = 12d^2 4^d d!^2$.

We first show that $R_{d,n}$ is a well-defined channel routing problem. In order to do this, we must show that q_i is well defined and that no pair of nets share an entry or exit column. Assume for the purposes of contradiction that q_i is not well defined for some i . Then there are two integers $s > r$ such that $i \equiv 4^{s-1} \pmod{4^s s!^2}$ and $i \equiv 4^{r-1} \pmod{4^r r!^2}$. The first congruence implies that 4^{s-1} divides i and thus that 4^r divides i (since $s-1 \geq r$) which clearly contradicts the second congruence. Thus q_i is well defined for each i .

Next assume (for the purposes of contradiction) that two nets have the same entry or exit column. Since it is easily seen that no two nets can have the same entry column, this means that there are two integers $i < j$ such that $q_i = q_j$. Since $q_j \geq j$, it is clear that $q_i \neq i$ and thus that $q_i = i + 4^s s!^2$ where $i \equiv 4^{s-1} \pmod{4^s s!^2}$ for some s . Thus $q_j = q_i \equiv 4^{s-1} \pmod{4^s s!^2}$ and it is clear that $j \neq q_j$. This means that $q_j = j + 4^r r!^2$ where $j \equiv 4^{r-1} \pmod{4^r r!^2}$ for some r and thus that $q_j \equiv 4^{r-1} \pmod{4^r r!^2}$. Since $q_i = q_j$, we can conclude by the arguments of the previous paragraph that $r = s$ and thus that $i = j$, a contradiction.

It is also easy to show that $R_{d,n}$ has density d . This is due to the fact that the nontrivial nets of $R_{d,n}$ can be partitioned into blocks B_1, \dots, B_d where

$$B_s = \{(4^{s-1}, 4^{s-1} + 4^s s!^2), (4^{s-1} + 4^s s!^s, 4^{s-1} + 2 \cdot 4^s s!^2), \dots, \\ (4^{s-1} + n - 4^s s!^2, 4^{s-1} + n)\}$$

for $1 \leq s \leq d$. Each vertical cut of the channel is crossed by at most one net from each block and thus the density of $R_{d,n}$ is at most d . In fact, the density is precisely d since any cut in the range $4^{d-1} < x < n + 1$ is crossed by a net from every block.

Note that the nets in any block B_s correspond to a 1-shift that is spread out across the channel. Hence our lower bound construction consists only of a collection of interlaced 1-shifts and trivial nets.

3. Proof of the Lower Bound

The proof that $R_{d,n}$ requires channel width $2d - 1$ proceeds in two parts. In Part 1, we show that wires passing through a large region of the channel must be routed in a highly restricted manner, and that there is such a region that contains the solution to a subproblem of $R_{d,n}$, which is isomorphic to $R_{d,f(d)}$ where $f(d) = 4^d d!^2$. In Part 2, we show by induction on d that any solution of $R_{d,f(d)}$ in such a region has width at least $2d - 1$.

3.1. Part 1

Assume for the purposes of contradiction that $R_{d,n}$ has a solution with width t where $t < 2d - 1$. Given any integer such that $0 < y < t$, let P_y be a path which travels between

columns n and $n + 1$ in rows 0 through y , then between rows y and $y + 1$ in columns n through 1 and finally between columns 1 and 0 in rows $y + 1$ to $t + 1$. For example, see Figure 3.

Precisely n nets must cross P_y at least once for any y . In what follows, we will be concerned primarily with the *initial crossing* of each wire on its route from the i th column of row 0 to the q_i th column of row $t + 1$. Since at most t wires can cross the vertical portions of P_y , at least $n - t$ wires must initially cross P_y on its horizontal portion. (Note that wires at such points are travelling in a *downward direction* across P_y .) Since n columns of the grid cross the horizontal portion of P_y , at most t of them can fail to contain an initial crossing of P_y . Summing over all paths P_y , we find that at most $t(t + 1) \leq 4d^2 - 2$ unit segments of columns 1 through n (those between row y and row $y + 1$ for some y) fail to contain a wire which is travelling in a downward direction. Thus there is some region of $(n/4d^2) = 3 \cdot 4^d d!^2$ consecutive columns for which every column segment contains a wire travelling in the *downward direction*. Henceforth, we will refer to this part of the solution as the *restricted region*.

Since the restricted region contains at least $3 \cdot 4^d d!^2$ consecutive columns, it is easily observed that it contains a subproblem which is isomorphic to $R_{d,f(d)}$ where $f(d) = 4^d d!^2$. This is because $4^{s_1} s_1!^2$ divides $4^{s_2} s_2!^2$ for whenever $s_1 < s_2$, and thus $R_{d,m}$ consists of consecutive blocks of $R_{d,f(d)}$ for any $m > f(d)$. This concludes Part 1.

3.2. Part 2

We now show that any solution to $R_{d,f(d)}$ in a restricted region requires $2d - 1$ tracks. The proof is by induction on d . This hypothesis is certainly true for $d = 1$. In what follows, we assume the hypothesis is true for $d - 1$ in order to verify it for d . First, however, we must establish some useful facts concerning routings in restricted regions. For instance, each row in such a region is either *full* (i.e., each unit segment of the row contains a wire) or *empty* (none of the unit segments contains a wire). This simple but powerful observation follows from the fact that every column in a restricted region is full. Thus a row which is neither full nor empty must contain a point which is incident to two column wire segments and one row wire segment. This is clearly impossible.

It is natural to group the rows together into (alternating) blocks of full rows and empty

rows. Such blocks are said to be *full* or *empty*, respectively. Notice that a wire can change layers only in an empty block and can change columns only in a full block. For example, see Figure 4.

Only a restricted set of column changes is possible in a full block of rows. For example, consider a full block that contains a long horizontal wire segment. Since we are routing in a restricted region, the columns crossing the wire must contain wires as in Figure 5a. Since all of the crossing wires are moving in a downward direction, no two can be connected by a unit horizontal wire, and thus the rows above and below the long horizontal wire also contain long horizontal wires (provided that they are not empty, of course). For example, see Figure 5b. By repeating this argument for the remaining rows, we can deduce that all the rows of the full block contain long horizontal wires. For example, see Figure 5c.

We are now ready to prove that any routing of $R_{d,f(d)}$ in a restricted region requires $2d-1$ tracks. For the purposes of contradiction, assume otherwise, and consider the net $N_i = (i, q_i)$ where $i = 4^{d-1}$ and $q_i = 4^{d-1} + 4^d d!^2$. Because we are in a restricted region, the corresponding wire in the solution must eventually travel from column 4^{d-1} to column $4^{d-1} + 4^d d!^2$ in a downward (or level) fashion. This means that some row of the solution contains a horizontal wire segment of length at least $4^d d!^2 / t \geq 4^d d!^2 / (2d-2)$. Thus, the block containing this row resembles that in Figure 5c. In particular, there is a subregion consisting of at least $\frac{4^d d!^2}{2d-2} - 2(2d-2)$ consecutive columns that is spanned by continuous horizontal wires in every row of the block. For $d > 1$,

$$\frac{4^d d!^2}{2d-2} - 2(2d-2) > (2d+2)f(d-1),$$

and thus the wires of at least $2d$ subproblems isomorphic to $R_{d-1,f(d-1)}$ enter and exit in the columns through which all of the long horizontal wire segments pass. Of these, at most $t \leq 2d-2$ can have a wire which passes outside those columns (since it must also re-enter on some other row). Thus the solution of at least one of the subproblems isomorphic to $R_{d-1,f(d-1)}$ is totally contained within the region spanned by the long horizontal wires. As the wires in this solution must pass straight through (downward) the block containing the long wires, the horizontal rows of the block can be removed without affecting the solution of $R_{d-1,f(d-1)}$. In addition, the neighboring empty blocks can be replaced by a single empty

ow, in which all of the appropriate layer changes can be made. The resulting solution for $R_{d-1, f(d-1)}$ must, by induction, have $2(d-1) - 1 = 2d - 3$ rows. Since the full block which was removed had at least one row and since at least one empty row was removed, we can conclude that the solution for $R_{d, f(d)}$ had at least $2d - 3 + 2 = 2d - 1$ rows. This concludes the proof.

4. Remarks

It is interesting to note that the construction consists mostly of trivial nets and does not contain any rising nets whatsoever. A priori, such a channel routing problem might have been considered to be easier than one which corresponds to a *permutation* (i.e., one in which every entry column is also an exit column). As we have seen, however, this is not the case. As one might expect, it is not difficult to modify the construction in order to prove an identical lower bound for channel routing problems which are permutations.

The term "trivial net" is a misnomer. For example, adding trivial nets to some channel routing problems (such as a 2-shift) increases their optimal channel width. In addition, the optimal routing of a trivial net can be quite complicated. For example, the obvious routing of a trivial net $N_i = (p_i, p_i)$ is to run a wire down the p_i th column from row 0 to row $t + 1$. Even if layer changes are allowed for such wires, we have constructed examples of $O(d^2)$ -net channel routing problems with optimal width $d + O(\log d)$ for which any such solution requires $2d - 1$ horizontal tracks.

Whether or not the number of wires in our lower bound example can be significantly decreased is an interesting open question. Although the size of our construction was artificially increased in order to simplify the proof, we do not know of any n -net channel routing problem with density d and optimal width $2d - 1$ for which $n \leq o(d!)$. Thus it is still possible that a channel routing algorithm could be found which produces solutions with width $d + \log n$.

A closer examination of our lower bound proof reveals that the restricted region of the routing contains at least d full rows and at least $d - 1$ empty rows. This coincides exactly with the behavior of the $(2d - 1)$ -track algorithms. Those algorithms use d tracks for routing and

$d-1$ tracks for layer changes. Hence the need for layer changes is a crucial factor in determining the channel width of problems in the 2-layer knock-knee model, and it cannot be ignored.

5. Acknowledgements

We are grateful to Gary Miller for mentioning this problem to us and for his assistance with this work. We would also like to thank Ron Rivest for his helpful comments.

6. References

1. Baker, B., S.N. Bhatt and T. Leighton, "An Approximation Algorithm for Manhattan Routing," pp. 205-229 in *Advances in Computing Research 2 (VLSI Theory)*, ed. F.P. Preparata, JAI Press, 1984.
2. Berger, B., M. Brady, D.J. Brown and F.T. Leighton, "Nearly Optimal Bounds and Algorithms for Multilayer Channel Routing," unpublished manuscript, 1986.
3. Bolognesi, T., and O. Brown, "A Channel Routing Algorithm with Bounded Wire Length," unpublished manuscript, CSL, U. Illinois, 1982.
4. Frank, A., "Disjoint Paths in a Rectilinear Grid," *Combinatorica*, Vol. 2, No. 4, pp. 361-371, 1982.
5. Gao, S., and S. Hambrusch, "Two-Layer Channel Routing with Vertical Unit-Length Overlap," *Algorithmica*, Vol. 1, No. 2, pp. 223-232, 1986.
6. Leighton, T., "A Survey of Bounds and Algorithms for 2-Sided Channel Routing," *Proc. of the Bonn Workshop on Paths, Flows and VLSI Layout*, 1988, to appear.
7. Mehlhorn, K., F. Preparata and M. Sarrafzadeh, "Channel Routing in Knock-Knee Mode: Simplified Algorithms and Proofs," Univ. Saarbrücken Technical Report, Nov. 1984.

8. Preparata, F.P., and W. Lipski, "Optimal Three-Layer Channel Routing," *IEEE Trans. on Computers C-33*, pp. 427-437, 1984.
9. Rivest, R.L., A. Barats and G. Miller, "Provably Good Channel Routing Algorithms," *1981 CMAU Conf. on VLSI Systems and Computations*, pp. 153-159, Oct. 1981.

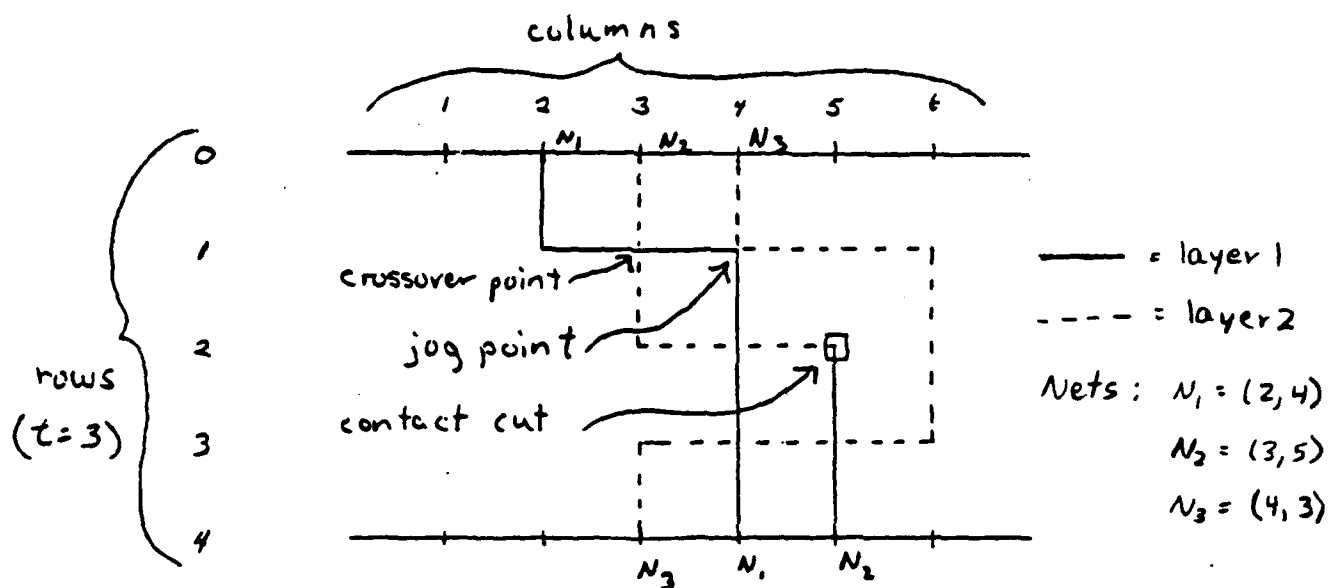


Figure 1. A solution to a channel routing problem using the knock-knee model.

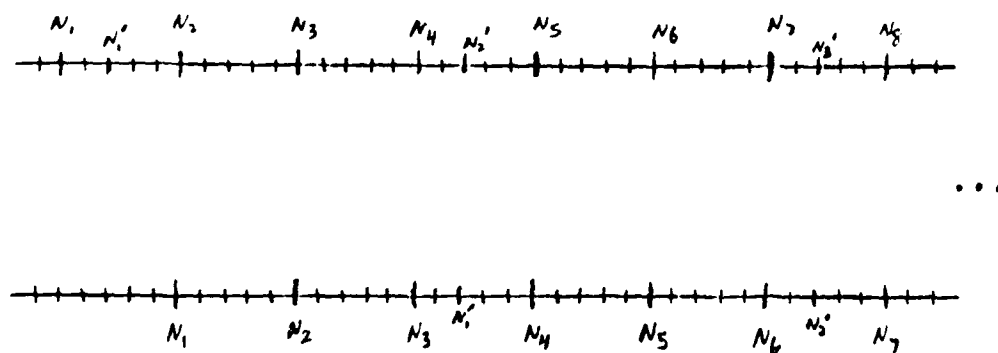


Figure 2. An informal drawing of the lower bound construction for $d = 2$. The problem consists mostly of trivial nets (which are not shown) along with a widely spaced 1-shift sequence of nets (N_1, N_2, N_3, \dots) and another even more widely spaced 1-shift sequence of nets (N'_1, N'_2, \dots) .

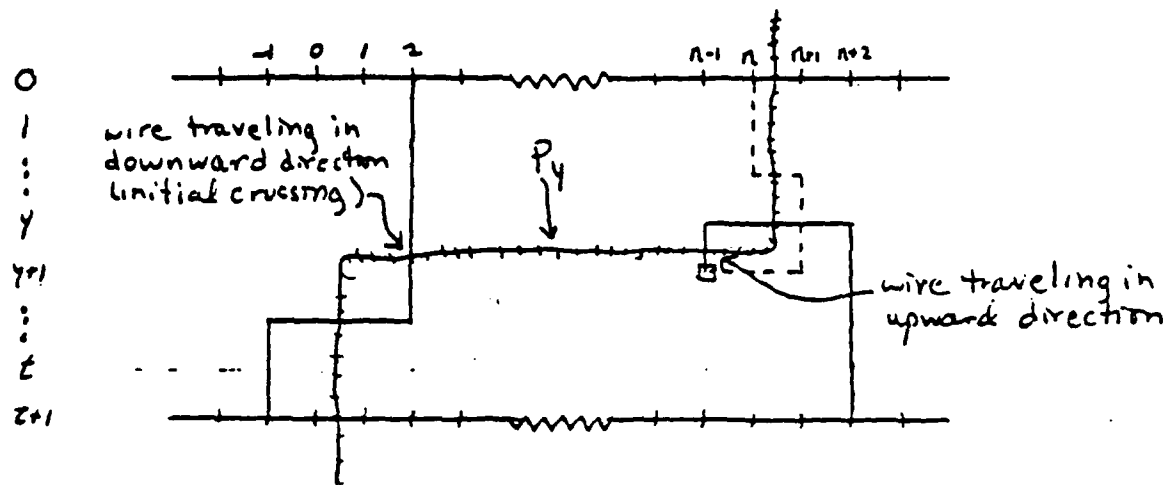


Figure 3. The path P_y .

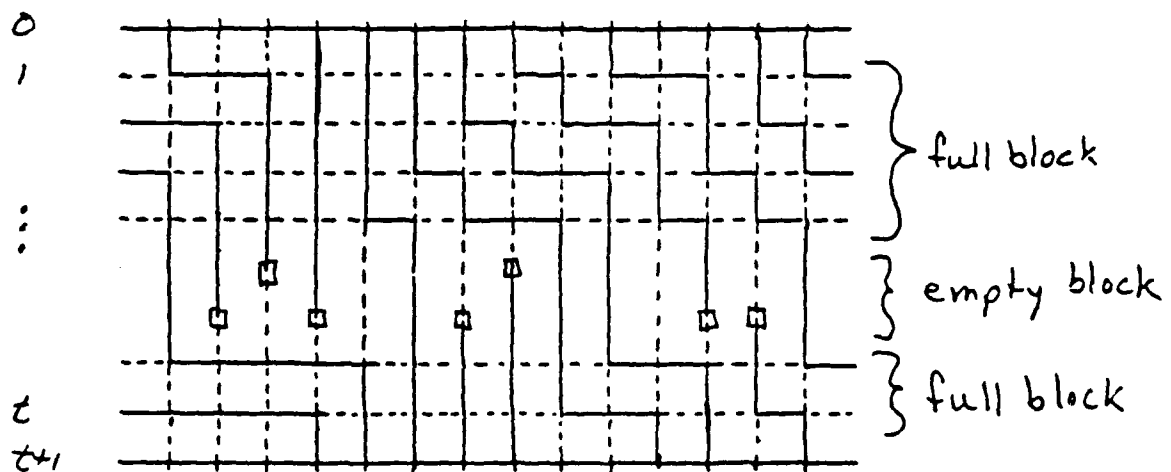


Figure 4. Wiring in a restricted region.

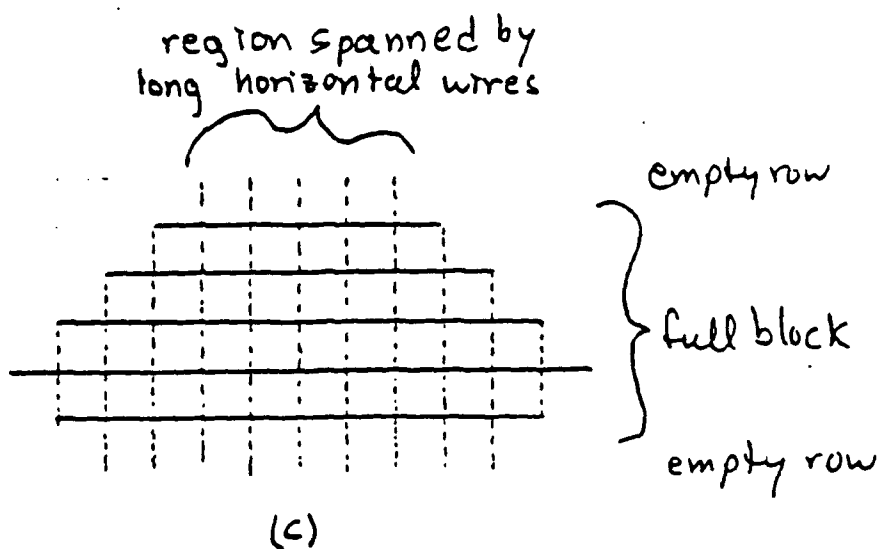
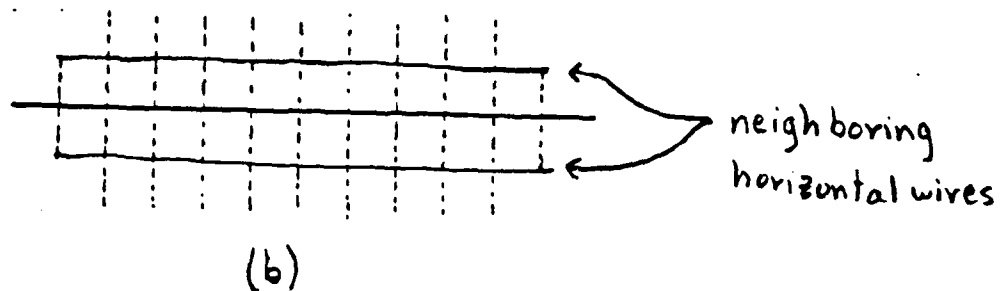
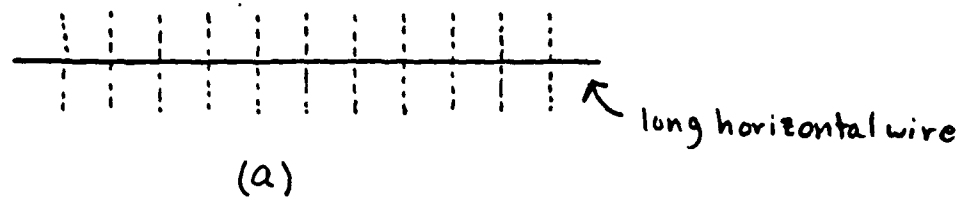


Figure 5. Impact of a long horizontal wire on a full block.